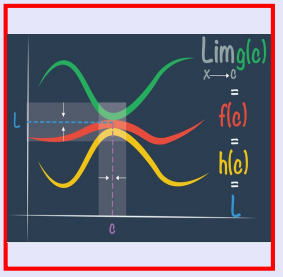
Calculus I

Lecture 15



Feb 19-8:47 AM

Class QZ 12 **Closed Notes**

Use linear approximation to estimate $\sqrt{27}$.

Box Your Final Ans.

$\sqrt{27} \approx \sqrt{25}$

$f(x) = \sqrt{x}$

$a = 25$

$f'(x) = \frac{1}{2\sqrt{x}}$

$f(x) \approx f(a) + f'(a)(x-a)$

$\sqrt{x} \approx \sqrt{25} + \frac{1}{2\sqrt{25}}(x-25)$

$\sqrt{x} \approx 5 + \frac{1}{10}(x-25)$

$\sqrt{27} \approx 5 + \frac{1}{10}(27-25)$

$= 5 + \frac{1}{10} \cdot 2$

$= 5 + \frac{1}{5} = \boxed{\frac{26}{5}} = \boxed{5.2}$

Jul 10-6:55 AM

$x^2 + y^2 + z^2 = 9$

$\frac{dy}{dt} = 4$, $\frac{dx}{dt} = 5$ Find $\frac{dz}{dt}$ when $(x, y, z) = (2, 2, 1)$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} + 2z\frac{dz}{dt} = 0$ Divide by 2 to reduce

$2 \cdot 5 + 2 \cdot 4 + 1\frac{dz}{dt} = 0$

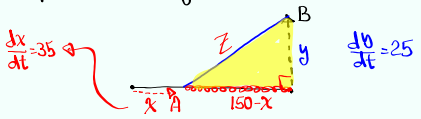
$18 + \frac{dz}{dt} = 0$ $\frac{dz}{dt} = -18$

z is decreasing

Jul 10-8:15 AM

At noon ship A is 150 Km west of ship B.

Ship A is sailing East at 35 Km/h while Ship B is sailing North at 25 Km/h.



How fast is the distance between them changing at 4:00 PM? $\frac{dz}{dt}$ at 4:00 PM

$(150-x)^2 + y^2 = z^2 \Rightarrow (150-140)^2 + 100^2 = z^2$

$z^2 = 10100$

$z = \sqrt{10100}$

$2(150-x)^1(-\frac{dx}{dt}) + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

Divide by 2 to reduce

$(150-x)(-35) + y(25) = z\frac{dz}{dt}$

$(150-140)(-35) + 100(25) = \sqrt{10100}\frac{dz}{dt}$

At 4:00 PM

4 hrs from noon

$x = 35(4) = 140$ $y = 25(4) = 100$

$d = r \cdot t$

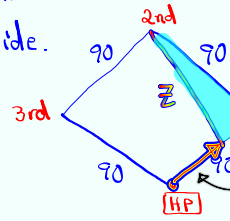
$-350 + 2500 = \sqrt{10100}\frac{dz}{dt}$

$\frac{dz}{dt} = \frac{2150}{\sqrt{10100}}$

$\frac{dz}{dt} \approx 21$ Km/h.

Jul 10-8:20 AM

A baseball diamond is a square of 90 ft Side.



$(90-x)^2+90^2=z^2$

A batter hits the ball, runs to 1st base with speed of 24 ft/s.

At what rate is his distance from second base decreasing when he is halfway to first base?

$x=45$

$2(90-x)^1\left(-\frac{dx}{dt}\right)+0=2z\frac{dz}{dt}$

$(90-45)(-24)=\sqrt{10125}\frac{dz}{dt}$

$45(-24)=\sqrt{10125}\frac{dz}{dt}$

$(90-45)^2+90^2=z^2$

$45^2+90^2=z^2$

$z^2=10125$

$z=\sqrt{10125}$

$\frac{dz}{dt}=\frac{-1080}{\sqrt{10125}}=-10.7$ ft/s

Jul 10-8:36 AM

Two sides of a triangle are 4m & 5m.

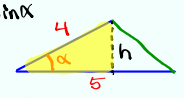
The angle between them is increasing at the rate of .06 rad/s.

At what rate is the area changing when the angle between the two fixed sides is $\frac{\pi}{3}$?

Area $=\frac{1}{2}bh$

$\sin\alpha=\frac{h}{4}$

$h=4\sin\alpha$



$b=5$

$h=4\sin\alpha$

$A=\frac{1}{2}\cdot5\cdot4\sin\alpha$

$A=10\sin\alpha$

$\frac{dA}{dt}=10\cos\alpha\frac{d\alpha}{dt}$

$\frac{dA}{dt}=10\cos\frac{\pi}{3}(.06)$

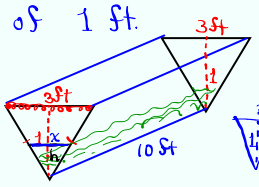
$=10\cdot\frac{1}{2}(.06)$

$=5(.06)$

$\frac{dA}{dt}=.3\ m^2/s$

Jul 10-8:47 AM

A trough is 10 ft long and its ends have the shape of isosceles triangles that are 3 ft across at the top and height of 1 ft.



$\frac{h}{x} = \frac{1}{3}$ $x = 3h$

It is being filled with water at the rate of 12 ft^3/min. $\frac{dV}{dt} = 12$

How fast is the water level increasing when the water is 6 inches deep? 12 inches = 1 ft

6 inches = .5 ft

V = Area of base · height

$= \frac{1}{2} b \cdot h \cdot 10$

$V = 5 \times h$

$V = 5 \cdot 3h \, h$

$V = 15h^2$

$\frac{dV}{dt} = 15 \cdot 2h \frac{dh}{dt}$

$12 = 30(.5) \frac{dh}{dt}$

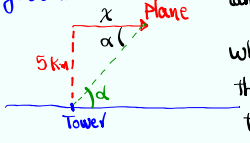
$12 = 15 \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{12}{15} = \frac{4}{5} = .8$

.8 ft/min

Jul 10-9:00 AM

A plane is flying horizontally at an altitude of 5Km and passes over tower on the ground



$\tan\alpha = \frac{5}{x}$ $x \tan\alpha = 5$

When angle of elevation is $\frac{\pi}{3}$, the angle is decreasing at the rate of $\frac{\pi}{6}$ Rad/min

$\frac{d\alpha}{dt} = -\frac{\pi}{6}$

How fast is the plane traveling at that time? $\frac{dx}{dt} = ?$

When $\alpha = \frac{\pi}{3}$

$\tan\frac{\pi}{3} = \sqrt{3}$

$\sec^2\frac{\pi}{3} = 4$

$x \tan\frac{\pi}{3} = 5$

$x = \frac{5}{\sqrt{3}}$

$x \tan\alpha = 5$

$\frac{dx}{dt} \cdot \tan\alpha + x \cdot \sec^2\alpha \cdot \frac{d\alpha}{dt} = 0$

$\frac{dx}{dt} \cdot \sqrt{3} + \frac{5}{\sqrt{3}} \cdot 4 \cdot -\frac{\pi}{6} = 0$

Multiply by $6\sqrt{3}$

$6\sqrt{3}\frac{dx}{dt} \cdot \sqrt{3} + 6\sqrt{3} \cdot \frac{5}{\sqrt{3}} \cdot 4 \cdot -\frac{\pi}{6} = 0$

$18\frac{dx}{dt} - 20\pi = 0$

$\frac{dx}{dt} = \frac{20\pi}{18} = \frac{10\pi}{9}$ Km/min ≈ 3.5 Km/min.

Convert to hr

≈ 10 Km/hr

final Ans.

Jul 10-9:30 AM